

1 bprobit: Bivariate Logistic Regression for Two Dichotomous Dependent Variables

Use the bivariate probit regression model if you have two binary dependent variables (Y_1, Y_2), and wish to model them jointly as a function of some explanatory variables. Each pair of dependent variables (Y_{i1}, Y_{i2}) has four potential outcomes, ($Y_{i1} = 1, Y_{i2} = 1$), ($Y_{i1} = 1, Y_{i2} = 0$), ($Y_{i1} = 0, Y_{i2} = 1$), and ($Y_{i1} = 0, Y_{i2} = 0$). The joint probability for each of these four outcomes is modeled with three systematic components: the marginal $\Pr(Y_{i1} = 1)$ and $\Pr(Y_{i2} = 1)$, and the correlation parameter ρ for the two marginal distributions. Each of these systematic components may be modeled as functions of (possibly different) sets of explanatory variables.

1.0.1 Syntax

```
> z.out <- zelig(list(mu1 = Y1 ~ X1 + X2,
                    mu2 = Y2 ~ X1 + X3,
                    rho = ~ 1),
                model = "bprobit", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out)
```

1.0.2 Input Values

In every bivariate probit specification, there are three equations which correspond to each dependent variable (Y_1, Y_2), and the correlation parameter ρ . Since the correlation parameter does not correspond to one of the dependent variables, the model estimates ρ as a constant by default. Hence, only two formulas (for μ_1 and μ_2) are required. If the explanatory variables for μ_1 and μ_2 are the same and effects are estimated separately for each parameter, you may use the following short hand:

```
> fml <- list(cbind(Y1, Y2) ~ X1 + X2)
```

which has the same meaning as:

```
> fml <- list(mu1 = Y1 ~ X1 + X2,
+            mu2 = Y2 ~ X1 + X2,
+            rho = ~ 1)
```

You may use the function `tag()` to constrain variables across equations. The `tag()` function takes a variable and a label for the effect parameter. Below, the constrained effect of `x3` in both equations is called the `age` parameter:

```
> fml <- list(mu1 = y1 ~ x1 + tag(x3, "age"),
+            mu2 = y2 ~ x2 + tag(x3, "age"))
```

You may also constrain different variables across different equations to have the same effect.

1.0.3 Examples

1. Basic Example

Load the data and estimate the model:

```
> data(sanction)

> z.out1 <- zelig(cbind(import, export) ~ coop + cost + target,
+               model = "bprobit", data = sanction)
```

By default, `zelig()` estimates two effect parameters for each explanatory variable in addition to the correlation coefficient; this formulation is parametrically independent (estimating unconstrained effects for each explanatory variable), but stochastically dependent because the models share a correlation parameter.

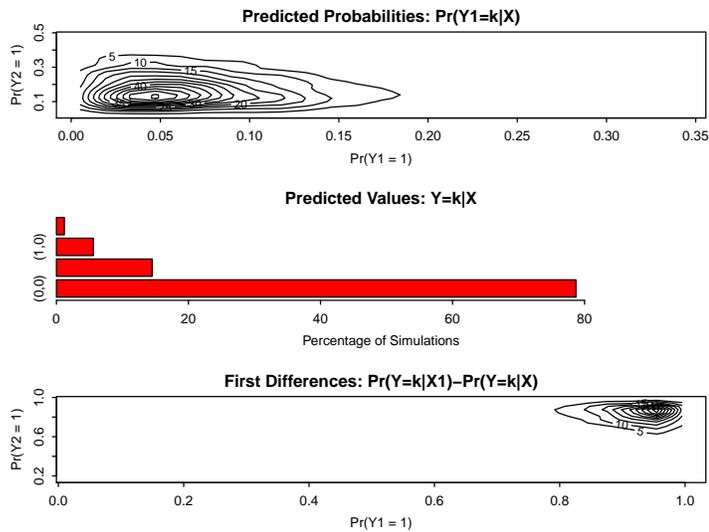
Generate baseline values for the explanatory variables (with cost set to 1, net gain to sender) and alternative values (with cost set to 4, major loss to sender):

```
> x.low <- setx(z.out1, cost = 1)
> x.high <- setx(z.out1, cost = 4)
```

Simulate fitted values and first differences:

```
> s.out1 <- sim(z.out1, x = x.low, x1 = x.high)
> summary(s.out1)

> plot(s.out1)
```



2. Joint Estimation of a Model with Different Sets of Explanatory Variables

Using the sample data `sanction`, estimate the statistical model, with `import` a function of `coop` in the first equation and `export` a function of `cost` and `target` in the second equation:

```
> fml2 <- list(mu1 = import ~ coop,
+             mu2 = export ~ cost + target)

> z.out2 <- zelig(fml2, model = "bprobit", data = sanction)
> summary(z.out2)
```

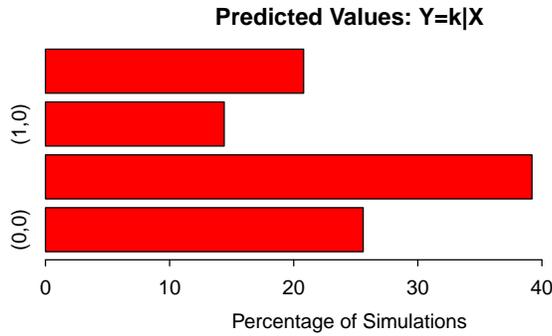
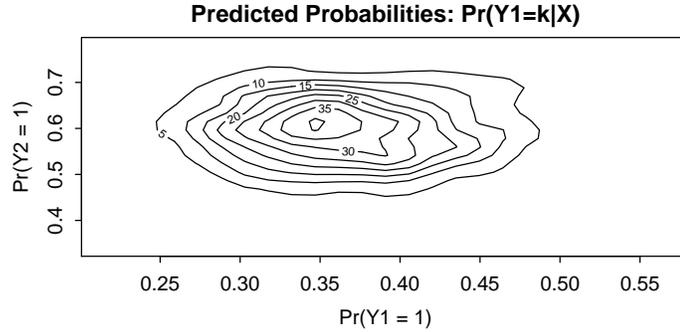
Set the explanatory variables to their means:

```
> x.out2 <- setx(z.out2)
```

Simulate draws from the posterior distribution:

```
> s.out2 <- sim(z.out2, x = x.out2)
> summary(s.out2)

> plot(s.out2)
```



1.0.4 Model

For each observation, define two binary dependent variables, Y_1 and Y_2 , each of which take the value of either 0 or 1 (in the following, we suppress the observation index i). We model the joint outcome (Y_1, Y_2) using two marginal probabilities for each dependent variable, and the correlation parameter, which describes how the two dependent variables are related.

- The *stochastic component* is described by two latent (unobserved) continuous variables which follow the bivariate Normal distribution:

$$\begin{pmatrix} Y_1^* \\ Y_2^* \end{pmatrix} \sim N_2 \left\{ \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right\},$$

where μ_j is a mean for Y_j^* and ρ is a scalar correlation parameter. The following observation mechanism links the observed dependent variables, Y_j , with these latent variables

$$Y_j = \begin{cases} 1 & \text{if } Y_j^* \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

- The *systemic components* for each observation are

$$\begin{aligned}\mu_j &= x_j \beta_j \quad \text{for } j = 1, 2, \\ \rho &= \frac{\exp(x_3 \beta_3) - 1}{\exp(x_3 \beta_3) + 1}.\end{aligned}$$

1.0.5 Quantities of Interest

For n simulations, expected values form an $n \times 4$ matrix.

- The expected values (**qi\$ev**) for the binomial probit model are the predicted joint probabilities. Simulations of β_1 , β_2 , and β_3 (drawn from their sampling distributions) are substituted into the systematic components, to find simulations of the predicted joint probabilities $\pi_{rs} = \Pr(Y_1 = r, Y_2 = s)$:

$$\begin{aligned}\pi_{11} &= \Pr(Y_1^* \geq 0, Y_2^* \geq 0) = \int_0^\infty \int_0^\infty \phi_2(\mu_1, \mu_2, \rho) dY_2^* dY_1^* \\ \pi_{10} &= \Pr(Y_1^* \geq 0, Y_2^* < 0) = \int_0^\infty \int_{-\infty}^0 \phi_2(\mu_1, \mu_2, \rho) dY_2^* dY_1^* \\ \pi_{01} &= \Pr(Y_1^* < 0, Y_2^* \geq 0) = \int_{-\infty}^0 \int_0^\infty \phi_2(\mu_1, \mu_2, \rho) dY_2^* dY_1^* \\ \pi_{00} &= \Pr(Y_1^* < 0, Y_2^* < 0) = \int_{-\infty}^0 \int_{-\infty}^0 \phi_2(\mu_1, \mu_2, \rho) dY_2^* dY_1^*\end{aligned}$$

where r and s may take a value of either 0 or 1, ϕ_2 is the bivariate Normal density.

- The predicted values (**qi\$pr**) are draws from the multinomial distribution given the expected joint probabilities.
- The first difference (**qi\$fd**) in each of the predicted joint probabilities are given by

$$FD_{rs} = \Pr(Y_1 = r, Y_2 = s | x_1) - \Pr(Y_1 = r, Y_2 = s | x).$$

- The risk ratio (**qi\$rr**) for each of the predicted joint probabilities are given by

$$RR_{rs} = \frac{\Pr(Y_1 = r, Y_2 = s | x_1)}{\Pr(Y_1 = r, Y_2 = s | x)}.$$

- In conditional prediction models, the average expected treatment effect (**att.ev**) for the treatment group is

$$\frac{1}{\sum_{i=1}^n t_i} \sum_{i:t_i=1}^n \{Y_{ij}(t_i = 1) - E[Y_{ij}(t_i = 0)]\} \quad \text{for } j = 1, 2,$$

where t_i is a binary explanatory variable defining the treatment ($t_i = 1$) and control ($t_i = 0$) groups. Variation in the simulations are due to uncertainty in simulating $E[Y_{ij}(t_i = 0)]$, the counterfactual expected value of Y_{ij} for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_i = 0$.

- In conditional prediction models, the average predicted treatment effect (`att.pr`) for the treatment group is

$$\frac{1}{\sum_{i=1}^n t_i} \sum_{i:t_i=1}^n \left\{ Y_{ij}(t_i = 1) - Y_{ij}(\widehat{t_i = 0}) \right\} \text{ for } j = 1, 2,$$

where t_i is a binary explanatory variable defining the treatment ($t_i = 1$) and control ($t_i = 0$) groups. Variation in the simulations are due to uncertainty in simulating $Y_{ij}(\widehat{t_i = 0})$, the counterfactual predicted value of Y_{ij} for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_i = 0$.

1.0.6 Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run `z.out <- zelig(y ~ x, model = "bprobit", data)`, then you may examine the available information in `z.out` by using `names(z.out)`, see the `coefficients` by using `z.out$coefficients`, and obtain a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output object `z.out`, you may extract:
 - `coefficients`: the named vector of coefficients.
 - `fitted.values`: an $n \times 4$ matrix of the in-sample fitted values.
 - `predictors`: an $n \times 3$ matrix of the linear predictors $x_j \beta_j$.
 - `residuals`: an $n \times 3$ matrix of the residuals.
 - `df.residual`: the residual degrees of freedom.
 - `df.total`: the total degrees of freedom.
 - `rss`: the residual sum of squares.
 - `y`: an $n \times 2$ matrix of the dependent variables.
 - `zelig.data`: the input data frame if `save.data = TRUE`.
- From `summary(z.out)`, you may extract:
 - `coef3`: a table of the coefficients with their associated standard errors and t -statistics.

- `cov.unscaled`: the variance-covariance matrix.
- `pearson.resid`: an $n \times 3$ matrix of the Pearson residuals.
- From the `sim()` output object `s.out`, you may extract quantities of interest arranged as arrays indexed by simulation \times quantity \times `x`-observation (for more than one `x`-observation; otherwise the quantities are matrices). Available quantities are:
 - `qi$ev`: the simulated expected values (joint predicted probabilities) for the specified values of `x`.
 - `qi$pr`: the simulated predicted outcomes drawn from a distribution defined by the joint predicted probabilities.
 - `qi$fd`: the simulated first difference in the predicted probabilities for the values specified in `x` and `x1`.
 - `qi$rr`: the simulated risk ratio in the predicted probabilities for given `x` and `x1`.
 - `qi$att.ev`: the simulated average expected treatment effect for the treated from conditional prediction models.
 - `qi$att.pr`: the simulated average predicted treatment effect for the treated from conditional prediction models.

How to Cite the Bivariate Probit Model

Olivia Lau, Kosuke Imai, and Gary King. *Bivariate Probit Regression for Two Dichotomous Dependent Variables*, 2011

How to Cite the Zelig Software Package

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Zelig: Everyone’s Statistical Software,” <http://GKing.harvard.edu/zelig>.

Imai, Kosuke, Gary King, and Olivia Lau. (2008). “Toward A Common Framework for Statistical Analysis and Development.” *Journal of Computational and Graphical Statistics*, Vol. 17, No. 4 (December), pp. 892-913.

See also

The bivariate probit function is part of the VGAM package by Thomas Yee [3]. In addition, advanced users may wish to refer to `help(vglm)` in the VGAM library. Additional documentation is available at <http://www.stat.auckland.ac.nz/~yee.Sample> data are from [2]

References

- [1] Olivia Lau, Kosuke Imai, and Gary King. *Bivariate Probit Regression for Two Dichotomous Dependent Variables*, 2011.
- [2] Lisa Martin. *Coercive Cooperation: Explaining Multilateral Economic Sanctions*. Princeton University Press, 1992. Please inquire with Lisa Martin before publishing results from these data, as this dataset includes errors that have since been corrected.
- [3] T. W. Yee and T. J. Hastie. Reduced-rank vector generalized linear models. *Statistical Modelling*, 3:15–41, 2003.