

## 0.1 `ei.dynamic`: Quinn's Dynamic Ecological Inference Model

Given contingency tables with observed marginals, ecological inference (EI) models estimate each internal cell value for each table. Quinn's dynamic EI model estimates a dynamic Bayesian model for  $2 \times 2$  tables with temporal dependence across tables (units). The model is implemented using a Markov Chain Monte Carlo algorithm (via a combination of slice and Gibbs sampling). For a hierarchical Bayesian implementation of EI see Quinn's dynamic EI model (Section ??). For contingency tables larger than 2 rows by 2 columns, see  $R \times C$  EI (Section ??).

### Syntax

```
> z.out <- zelig(cbind(t0, t1) ~ x0 + x1, N = NULL,
                 model = "MCMCei.dynamic", data = mydata)
> x.out <- setx(z.out, fn = NULL, cond = TRUE)
> s.out <- sim(z.out, x = x.out)
```

### Inputs

- **t0, t1**: numeric vectors (either counts or proportions) containing the column marginals of the units to be analyzed.
- **x0, x1**: numeric vectors (either counts or proportions) containing the row marginals of the units to be analyzed.
- **N**: total counts in each contingency table (unit). If **t0, t1**, **x0** and **x1** are proportions, you must specify **N**.

### Additional Inputs

In addition, `zelig()` accepts the following additional inputs for `ei.dynamic` to monitor the convergence of the Markov chain:

- **burnin**: number of the initial MCMC iterations to be discarded (defaults to 5,000).
- **mcmc**: number of the MCMC iterations after burnin (defaults to 50,000).
- **thin**: thinning interval for the Markov chain. Only every **thin**-th draw from the Markov chain is kept. The value of **mcmc** must be divisible by this value. The default value is 1.
- **verbose**: defaults to **FALSE**. If **TRUE**, the progress of the sampler (every 10%) is printed to the screen.

- **seed**: seed for the random number generator. The default is **NA** which corresponds to a random seed of 12345.

The model also accepts the following additional arguments to specify priors and other parameters:

- **W**: a  $p \times p$  numeric matrix describing the structure of the temporal dependence among elements of  $\theta_0$  and  $\theta_1$ . The default value is 0, which constructs a weight matrix corresponding to random walk priors for  $\theta_0$  and  $\theta_1$  (assuming that the tables are equally spaced throughout time, and that the elements of **t0**, **t1**, **x0**, **x1** are temporally ordered).
- **a0**:  $a_0/2$  is the shape parameter for the Inverse Gamma prior on  $\sigma_0^2$ . The default is 0.825.
- **b0**:  $b_0/2$  is the scale parameter for the Inverse Gamma prior on  $\sigma_0^2$ . The default is 0.0105.
- **a1**:  $a_1/2$  is the shape parameter for the Inverse Gamma prior on  $\sigma_1^2$ . The default is 0.825.
- **b1**:  $b_1/2$  is the scale parameter for the Inverse Gamma prior on  $\sigma_1^2$ . The default is 0.0105.

Users may wish to refer to `help(MCMCdynamicEI)` for more options.

## Convergence

Users should verify that the Markov Chain converges to its stationary distribution. After running the `zelig()` function but before performing `setx()`, users may conduct the following convergence diagnostics tests:

- `geweke.diag(z.out$coefficients)`: The Geweke diagnostic tests the null hypothesis that the Markov chain is in the stationary distribution and produces z-statistics for each estimated parameter.
- `heidel.diag(z.out$coefficients)`: The Heidelberger-Welch diagnostic first tests the null hypothesis that the Markov Chain is in the stationary distribution and produces p-values for each estimated parameter. Calling `heidel.diag()` also produces output that indicates whether the mean of a marginal posterior distribution can be estimated with sufficient precision, assuming that the Markov Chain is in the stationary distribution.
- `raftery.diag(z.out$coefficients)`: The Raftery diagnostic indicates how long the Markov Chain should run before considering draws from the marginal posterior distributions sufficiently representative of the stationary distribution.

If there is evidence of non-convergence, adjust the values for `burnin` and `mcmc` and rerun `zelig()`.

Advanced users may wish to refer to `help(geeweke.diag)`, `help(heidel.diag)`, and `help(raftery.diag)` for more information about these diagnostics.

## Examples

### 1. Basic examples

Attaching the example dataset:

```
> data(eidat)
```

Estimating the model using `ei.dynamic`:

```
> z.out <- zelig(cbind(t0, t1) ~ x0 + x1, model = "ei.dynamic",  
+   data = eidat, mcmc = 40000, thin = 10, burnin = 10000, verbose = TRUE)  
> summary(z.out)
```

Setting values for in-sample simulations given the marginal values of `t0`, `t1`, `x0`, and `x1`:

```
> x.out <- setx(z.out, fn = NULL, cond = TRUE)
```

In-sample simulations from the posterior distribution:

```
> s.out <- sim(z.out, x = x.out)
```

Summarizing in-sample simulations at aggregate level weighted by the count in each unit:

```
> summary(s.out)
```

Summarizing in-sample simulations at unit level for the first 5 units:

```
> summary(s.out, subset = 1:5)
```

## Model

Consider the following  $2 \times 2$  contingency table for the racial voting example. For each geographical unit  $i = 1, \dots, p$ , the marginals  $t_i^0$ ,  $t_i^1$ ,  $x_i^0$ , and  $x_i^1$  are known, and we would like to estimate  $n_i^{00}$ ,  $n_i^{01}$ ,  $n_i^{10}$ , and  $n_i^{11}$ .

	No Vote	Vote	
Black	$n_i^{00}$	$n_i^{01}$	$x_i^0$
White	$n_i^{10}$	$n_i^{11}$	$x_i^1$
	$t_i^0$	$t_i^1$	$N_i$

The marginal values  $x_i^0$ ,  $x_i^1$ ,  $t_i^0$ ,  $t_i^1$  are observed as either counts or fractions. If fractions, the counts can be obtained by multiplying by the total counts per table  $N_i = n_i^{00} + n_i^{01} + n_i^{10} + n_i^{11}$ , and rounding to the nearest integer. Although there are four internal cells, only two unknowns are modeled since  $n_i^{01} = x_i^0 - n_i^{00}$  and  $n_i^{11} = x_i^1 - n_i^{10}$ .

The hierarchical Bayesian model for ecological inference in  $2 \times 2$  is illustrated as following:

- The *stochastic component* of the model assumes that

$$\begin{aligned} n_i^{00} \mid x_i^0, \beta_i^b &\sim \text{Binomial}(x_i^0, \beta_i^b), \\ n_i^{10} \mid x_i^1, \beta_i^w &\sim \text{Binomial}(x_i^1, \beta_i^w) \end{aligned}$$

where  $\beta_i^b$  is the fraction of the black voters who vote and  $\beta_i^w$  is the fraction of the white voters who vote.  $\beta_i^b$  and  $\beta_i^w$  as well as their aggregate summaries are the focus of inference.

- The *systematic component* of the model is

$$\begin{aligned} \beta_i^b &= \frac{\exp \theta_i^0}{1 - \exp \theta_i^0} \\ \beta_i^w &= \frac{\exp \theta_i^1}{1 - \exp \theta_i^1} \end{aligned}$$

The logit transformations of  $\beta_i^b$  and  $\beta_i^w$ ,  $\theta_i^0$ , and  $\theta_i^1$  now take value on the real line. (Future versions may allow  $\beta_i^b$  and  $\beta_i^w$  to be functions of observed covariates.)

- The *priors* for  $\theta_i^0$  and  $\theta_i^1$  are given by

$$\begin{aligned} \theta_i^0 \mid \sigma_0^2 &\propto \frac{1}{\sigma_0^p} \exp \left( -\frac{1}{2\sigma_0^2} \theta_0' P \theta_0 \right) \\ \theta_i^1 \mid \sigma_1^2 &\propto \frac{1}{\sigma_1^p} \exp \left( -\frac{1}{2\sigma_1^2} \theta_1' P \theta_1 \right) \end{aligned}$$

where  $P$  is a  $p \times p$  matrix whose off diagonal elements  $P_{ts}$  ( $t \neq s$ ) equal  $-W_{ts}$  (the negative values of the corresponding elements of the weight matrix  $W$ ), and diagonal elements  $P_{tt} = \sum_{s \neq t} W_{ts}$ . Scale parameters  $\sigma_0^2$  and  $\sigma_1^2$  have hyperprior distributions as given below.

- The *hyperpriors* for  $\sigma_0^2$  and  $\sigma_1^2$  are given by

$$\begin{aligned}\sigma_0^2 &\sim \text{Inverse Gamma} \left( \frac{a_0}{2}, \frac{b_0}{2} \right), \\ \sigma_1^2 &\sim \text{Inverse Gamma} \left( \frac{a_1}{2}, \frac{b_1}{2} \right),\end{aligned}$$

where  $a_0/2$  and  $a_1/2$  are the shape parameters of the (independent) Gamma distributions while  $b_0/2$  and  $b_1/2$  are the scale parameters.

The default hyperpriors for  $\mu_0$ ,  $\mu_1$ ,  $\sigma_0^2$ , and  $\sigma_1^2$  are chosen such that the prior distributions for  $\beta^b$  and  $\beta^w$  are flat.

## Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run:

```
> z.out <- (cbind(t0, t1) ~ x0 + x1, N = NULL,
            model = "ei.dynamic", data = mydata)
```

then you may examine the available information in `z.out` by using `names(z.out)`, see the draws from the posterior distribution of the quantities of interest by using `z.out$coefficients`, and view a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output object `z.out`, you may extract:
  - **coefficients**: draws from the posterior distributions of the parameters.
  - **data**: the name of the input data frame.
  - **N**: the total counts when the inputs are fractions.
  - **seed**: the random seed used in the model.
- From `summary(z.out)`, you may extract:
  - **summary**: a matrix containing the summary information of the posterior estimation of  $\beta_i^b$  and  $\beta_i^w$  for each unit and the parameters  $\mu_0$ ,  $\mu_1$ ,  $\sigma_1$  and  $\sigma_2$  based on the posterior distribution. The first  $p$  rows correspond to  $\beta_i^b$ ,  $i = 1, \dots, p$ , the row

names are in the form of `p0tablei`. The  $(p + 1)$ -th to the  $2p$ -th rows correspond to  $\beta_i^w$ ,  $i = 1, \dots, p$ . The row names are in the form of `p1tablei`. The last four rows contain information about  $\mu_0$ ,  $\mu_1$ ,  $\sigma_0^2$  and  $\sigma_1^2$ , the prior means and variances of  $\theta_0$  and  $\theta_1$ .

- From the `sim()` output object `s.out`, you may extract quantities of interest arranged as arrays indexed by simulation  $\times$  column  $\times$  row  $\times$  observation, where column and row refer to the column dimension and the row dimension of the ecological table, respectively. In this model, only  $2 \times 2$  contingency tables are analyzed, hence column= 2 and row= 2 in all cases. Available quantities are:
  - `qi$ev`: the simulated expected values of each internal cell given the observed marginals.
  - `qi$pr`: the simulated expected values of each internal cell given the observed marginals.

## How to Cite

To cite the *ei.dynamic* Zelig model use:

Ben Goodrich and Ying Lu. 2007. “ei.dynamic: Quinn’s Dynamic Ecological Inference,” in Kosuke Imai, Gary King, and Olivia Lau, “Zelig: Everyone’s Statistical Software,” <http://gking.harvard.edu/zelig>.

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Zelig: Everyone’s Statistical Software,” <http://GKing.harvard.edu/zelig>.

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Toward A Common Framework for Statistical Analysis and Development,” <http://gking.harvard.edu/files/abs/z-abs.shtml>.

## See also

*ei.dynamic* function is part of the MCMCpack library by Andrew D. Martin and Kevin M. Quinn (Martin and Quinn 2005). The convergence diagnostics are part of the CODA library by Martyn Plummer, Nicky Best, Kate Cowles, and Karen Vines (Plummer et al. 2005). Sample data are adapted from Martin and Quinn (2005).

# Bibliography

Martin, A. D. and Quinn, K. M. (2005), *MCMCpack: Markov chain Monte Carlo (MCMC) Package*.

Plummer, M., Best, N., Cowles, K., and Vines, K. (2005), *coda: Output analysis and diagnostics for MCMC*.