0.1 `normal.net`: Network Normal Regression for Continuous Proximity Matrix Dependent Variables

The Network Normal regression model is a close variant of the more standard least squares regression model (see `netlm`). Both models specify a continuous proximity matrix (a.k.a. sociomatricies, adjacency matrices, or matrix representations of directed graphs) dependent variable as a linear function of a set of explanatory variables. The network Normal model reports maximum likelihood (rather than least squares) estimates. The two models differ only in their estimate for the stochastic parameter σ.

Syntax

```
> z.out <- zelig(y ~ x1 + x2, model = "normal.net", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out)
```

Additional Inputs

In addition to the standard inputs, `zelig()` takes the following additional options for network normal regression:

- LF: specifies the link function to be used for the network normal regression. Default is LF="identity", but LF can also be set to "log" or "inverse" by the user.

Examples

1. Basic Example

   Load the sample data (see `?friendship` for details on the structure of the network dataframe):

   ```
   > data(friendship)
   ```

   Estimate model:

   ```
   > z.out <- zelig(perpower ~ friends + advice + prestige, model = "normal.net",
   +     data = friendship)
   > summary(z.out)
   ```

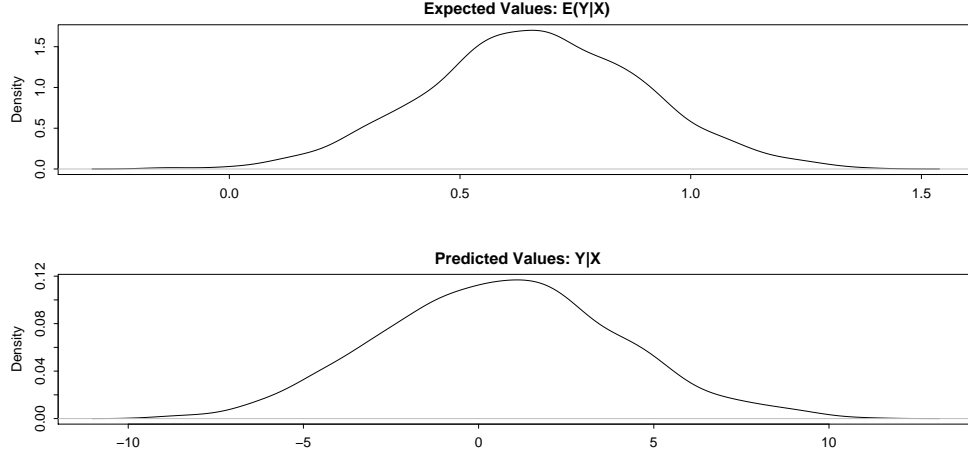   Setting values for the explanatory variables to their default values:

   ```
   > x.out <- setx(z.out)
   ```

   Simulate fitted values.

```
> s.out <- sim(z.out, x = x.out)
> summary(s.out)
> plot(s.out)
```


Model

The `normal.net` model performs a Normal regression of the proximity matrix $\mathbf{Y}$, a $m \times m$ matrix representing network ties, on a set of proximity matrices $\mathbf{X}$. This network regression model is directly analogous to standard Normal regression element-wise on the appropriately vectorized matrices. Proximity matrices are vectorized by creating Y, a $m^2 \times 1$ vector to represent the proximity matrix. The vectorization which produces the Y vector from the $\mathbf{Y}$ matrix is performed by simple row-concatenation of $\mathbf{Y}$. For example, if $\mathbf{Y}$ is a 15×15 matrix, the $\mathbf{Y}_{1,1}$ element is the first element of Y, and the $\mathbf{Y}_{2,1}$ element is the second element of Y and so on. Once the input matrices are vectorized, standard Normal regression is performed.

Let Y_i be the continuous dependent variable, produced by vectorizing a continuous proximity matrix, for observation i.

- The *stochastic component* is described by a univariate normal model with a vector of means μ_i and scalar variance σ^2:

$$Y_i \sim \mathrm{Normal}(\mu_i, \sigma^2).$$

- The *systematic component* is given by:

$$\mu_i = x_i\beta.$$

where x_i is the vector of k explanatory variables and β is the vector of coefficients.

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Quantities of Interest

The quantities of interest for the network Normal regression are the same as those for the standard Normal regression.

- The expected value (`qi$ev`) for the `normal.net` model is the mean of simulations from the stochastic component,
$$E(Y) = \mu_i = x_i\beta,$$
given a draw of β from its posterior.

- The predicted value (`qi$pr`) is a draw from the distribution defined by the set of parameters (μ_i, σ^2).

- The first difference (`qi$fd`) for the network Normal model is defined as
$$FD = \Pr(Y|x_1) - \Pr(Y|x)$$

Output Values

The output of each Zelig command contains useful information which you may view. For example, you run `z.out <- zelig(y ~ x, model = "normal.net", data)`, then you may examine the available information in `z.out` by using `names(z.out)`, see the coefficients by using `z.out$coefficients`, and a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output stored in `z.out`, you may extract:

  - `coefficients`: parameter estimates for the explanatory variables.
  - `fitted.values`: the vector of fitted values for the systemic component λ.
  - `residuals`: the working residuals in the final iteration of the IWLS fit.
  - `linear.predictors`: fitted values. For the normal model, these are identical to fitted values.
  - `aic`: Akaike's Information Criterion (minus twice the maximized log-likelihood plus twice the number of coefficients).
  - `bic`: the Bayesian Information Criterion (minus twice the maximized log-likelihood plus the number of coefficients times log n).
  - `df.residual`: the residual degrees of freedom.
  - `df.null`: the residual degrees of freedom for the null model.
  - `zelig.data`: the input data frame if `save.data = TRUE`

- From `summary(z.out)` (as well as from `zelig()`), you may extract:

  - `mod.coefficients`: the parameter estimates with their associated standard errors, p-values, and t statistics.

- cov.scaled: a $k \times k$ matrix of scaled covariances.
- cov.unscaled: a $k \times k$ matrix of unscaled covariances.

- From the sim() output stored in s.out, you may extract:
  - qi$ev: the simulated expected probabilities for the specified values of x.
  - qi$pr: the simulated predicted values drawn from the distribution defined by (μ_i, σ^2).
  - qi$fd: the simulated first differences in the expected probabilities simulated from x and x1.

How to Cite

To cite the *normal.net* Zelig model:

> Skyler J. Cranmer. 2007. "normal.net: Network Normal Regression for Continuous Proximity Matrix Dependent Variables," in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software," http://gking.harvard.edu/zelig.

To cite Zelig as a whole, please reference these two sources:

> Kosuke Imai, Gary King, and Olivia Lau. 2007. "Zelig: Everyone's Statistical Software," http://GKing.harvard.edu/zelig.

> Kosuke Imai, Gary King, and Olivia Lau. 2007. "Toward A Common Framework for Statistical Analysis and Development," http://gking.harvard.edu/files/abs/z-abs.shtml.

See also

The network normal regression is part of the netglm package by Skyler J. Cranmer and is built using some of the functionality of the sna package by Carter T. Butts (Butts and Carley 2001).In addition, advanced users may wish to refer to help(normal.net). Sample data are fictional.

Bibliography

Butts, C. and Carley, K. (2001), "Multivariate Methods for Interstructural Analysis," Tech. rep., CASOS working paper, Carnegie Mellon University.