

## 0.1 sur: Seemingly Unrelated Regression

`sur` extends ordinary least squares analysis to estimate system of linear equations with correlated error terms. The seemingly unrelated regression model can be viewed as a special case of generalized least squares.

### Syntax

```
> fml <- list ("mu1" = Y1 ~ X1,
              "mu2" = Y2 ~ X2,
              "mu3" = Y3 ~ X3)
> z.out<-zelig(formula = fml, model = "2sls", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out)
```

### Inputs

`sur` regression specification has at least  $M$  equations ( $M \geq 2$ ) corresponding to the dependent variables ( $Y_1, Y_2, \dots, Y_M$ ).

- **formula**: a list whose elements are formulae corresponding to the  $M$  equations and their respective dependent and explanatory variables. For example, when there are no constraints on the coefficients:

```
> fml <- list(mu1 = Y1 ~ X1, mu2 = Y2 ~ X2, mu3 = Y3 ~ X3)
```

"mu1" is the label for the first equation with  $Y_1$  as the dependent variable and  $X_1$  as the explanatory variable. Similarly "mu2" and "mu3" are the labels for the  $Y_2$  and  $Y_3$  equations.

- **tag**: Users can also put constraints on the coefficients by using the special function `tag`. `tag` takes two parameters. The first parameter is the variable whose coefficient needs to be constrained and the second parameter is label for the constrained coefficient. Each label uniquely identifies the constrained coefficient. For example:

```
> fml <- list(mu1 = Y1 ~ tag(Xc, "constrain1") + X1, mu2 = Y2 ~
+           tag(Xc, "constrain1") + X2, mu3 = Y3 ~ X3)
```

### Additional Inputs

`sur` takes the following additional inputs for model specifications:

- **TX**: an optional matrix to transform the regressor matrix and, hence, also the coefficient vector (see details). Default is `NULL`.
- **maxiter**: maximum number of iterations.

- `tol`: tolerance level indicating when to stop the iteration.
- `rcovformula`: formula to calculate the estimated residual covariance matrix (see details). Default is equal to 1.
- `probdfsys`: use the degrees of freedom of the whole system (in place of the degrees of freedom of the single equation to calculate probability values for the t-test of individual parameters.
- `solveto1`: tolerance level for detecting linear dependencies when inverting a matrix or calculating a determinant. Default is `solveto1=.Machine\$.double.eps`.
- `saveMemory`: logical. Save memory by omitting some calculation that are not crucial for the basic estimate (e.g McElroy's  $R^2$ ).

## Details

The matrix `TX` transforms the regressor matrix ( $X$ ) by  $X^* = X \times TX$ . Thus, the vector of coefficients is now  $b = TX \times b^*$  where  $b$  is the original(stacked) vector of all coefficients and  $b^*$  is the new coefficient vector that is estimated instead. Thus, the elements of vector  $b$  and  $b_i = \sum_j TX_{ij} \times b_{j^*}$ . The  $TX$  matrix can be used to change the order of the coefficients and also to restrict coefficients (if  $TX$  has less columns than it has rows). If iterated (with `maxit>1`), the convergence criterion is

$$\sqrt{\frac{\sum_i (b_{i,g} - b_{i,g-1})^2}{\sum_i b_{i,g-1}^2}} < tol$$

where  $b_{i,g}$  is the  $i$ th coefficient of the  $g$ th iteration step. The formula (`rcovformula` to calculate the estimated covariance matrix of the residuals( $\hat{\Sigma}$ ) can be one of the following (see Judge et al., 1955, p.469): if `rcovformula= 0`:

$$\hat{\sigma}_{ij} = \frac{\hat{e}_i \hat{e}_j}{T}$$

if `rcovformula= 1` or `rcovformula='geomean'`:

$$\hat{\sigma}_{ij} = \frac{\hat{e}_i \hat{e}_j}{\sqrt{(T - k_i) \times (T - k_j)}}$$

if `rcovformula= 2` or `rcovformula='Theil'`:

$$\hat{\sigma}_{ij} = \frac{\hat{e}_i \hat{e}_j}{T - k_i - k_j + tr[X_i(X_i'X_i)^{-1}X_i'X_j(X_j'X_j)^{-1}X_j']}$$

if `rcovformula= 3` or `rcovformula='max'`:

$$\hat{\sigma}_{ij} = \frac{\hat{e}_i' \hat{e}_j}{T - \max(k_i, k_j)}$$

If  $i = j$ , formula 1, 2, and 3 are equal. All these three formulas yield unbiased estimators for the diagonal elements of the residual covariance matrix. If *ineqj*, only formula 2 yields an unbiased estimator for the residual covariance matrix, but it is not necessarily positive semidefinite. Thus, it is doubtful whether formula 2 is really superior to formula 1 (Theil, 1971, p.322).

## Examples

Attaching the example dataset:

```
> data(grunfeld)
```

Formula:

```
> formula <- list(mu1 = Ige ~ Fge + Cge, mu2 = Iw ~ Fw + Cw)
```

Estimating the model using `sur`:

```
> z.out <- zelig(formula = formula, model = "sur", data = grunfeld)
```

```
> summary(z.out)
```

Set explanatory variables to their default (mean/mode) values

```
> x.out <- setx(z.out)
```

Simulate draws from the posterior distribution:

```
> s.out <- sim(z.out, x = x.out)
```

```
> summary(s.out)
```

## Model

The basic seemingly unrelated regression model assumes that for each individual observation  $i$  there are  $M$  dependent variables ( $Y_{ij}, j = 1, \dots, M$ ) each with its own regression equation:

$$Y_{ij} = X'_{ij}\beta_j + \epsilon_{ij}, \quad \text{for } i = 1, \dots, N \quad \text{and} \quad j = 1, \dots, M$$

when  $X_{ij}$  is a  $k$ -vector of explanatory variables,  $\beta_j$  is the coefficients of the explanatory variables,

- The *stochastic component* is:

$$\epsilon_{ij} \sim \mathcal{N}(0, \sigma_{ij})$$

where within each  $j$  equation,  $\epsilon_{ij}$  is identically and independently distributed for  $i = 1, \dots, N$ ,

$$\text{Var}(\epsilon_{ij}) = \sigma_j \quad \text{and} \quad \text{Cov}(\epsilon_{ij}, \epsilon_{i'j}) = 0, \quad \text{for } i \neq i', \quad \text{and } j = 1, \dots, M$$

However, the error terms for the  $i$ th observation can be correlated across equations

$$\text{Cov}(\epsilon_{ij}, \epsilon_{ij'}) \neq 0, \quad \text{for } j \neq j', \quad \text{and } i = 1, \dots, N$$

- The *systematic component* is:

$$\mu_{ij} = E(Y_{ij}) = X_{ij}\beta_j, \quad \text{for } i = 1, \dots, N, \quad \text{and } j = 1, \dots, M$$

## See Also

For information about two stage least squares regression, see Section ?? and `help(2sls)`. For information about three stage least squares regression, see Section ?? and `help(3sls)`.

## Quantities of Interest

### Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run:

```
z.out <- zelig(formula=fml, model = "sur", data)
```

then you may examine the available information in `z.out` by using `names(z.out)`, see the draws from the posterior distribution of the coefficients by using `z.out$coefficients`, and view a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below:

- `rcovest`: residual covariance matrix used for estimation.

- `mcelr2`: McElroys R-squared value for the system.
- `method`: Estimation method.
- `g`: number of equations.
- `n`: total number of observations.
- `k`: total number of coefficients.
- `ki`: total number of linear independent coefficients.
- `df`: degrees of freedom of the whole system.
- `iter`: number of iteration steps.
- `b`: vector of all estimated coefficients.
- `t`:  $t$  values for  $b$ .
- `se`: estimated standard errors of  $b$ .
- `bt`: coefficient vector transformed by  $TX$ .
- `p`:  $p$  values for  $b$ .
- `bcov`: estimated covariance matrix of  $b$ .
- `btcov`: covariance matrix of  $bt$ .
- `rcov`: estimated residual covariance matrix.
- `drcov`: determinant of `rcov`.
- `rcor`: estimated residual correlation matrix.
- `olsr2`: system OLS R-squared value.
- `y`: vector of all (stacked) endogenous variables.
- `x`: matrix of all (diagonally stacked) regressors.
- `data`: data frame of the whole system (including instruments).
- `TX`: matrix used to transform the regressor matrix.
- `rcovformula`: formula to calculate the estimated residual covariance matrix.
- `probdfsys`: system degrees of freedom to calculate probability values?.
- `solveto1`: tolerance level when inverting a matrix or calculating a determinant.

- **eq**: a list that contains the results that belong to the individual equations.
- **eqnlabel\***: the equation label of the *i*th equation (from the labels list).
- **formula\***: model formula of the *i*th equation.
- **n\***: number of observations of the *i*th equation.
- **k\***: number of coefficients/regressors in the *i*th equation (including the constant).
- **ki\***: number of linear independent coefficients in the *i*th equation (including the constant differs from *k* only if there are restrictions that are not cross equation).
- **df\***: degrees of freedom of the *i*th equation.
- **b\***: estimated coefficients of the *i*th equation.
- **se\***: estimated standard errors of *b* of the *i*th equation.
- **t\***: *t* values for *b* of the *i*th equation.
- **p\***: *p* values for *b* of the *i*th equation.
- **covb\***: estimated covariance matrix of *b* of the *i*th equation.
- **y\***: vector of endogenous variable (response values) of the *i*th equation.
- **x\***: matrix of regressors (model matrix) of the *i*th equation.
- **data\***: data frame (including instruments) of the *i*th equation.
- **fitted\***: vector of fitted values of the *i*th equation.
- **residuals\***: vector of residuals of the *i*th equation.
- **ssr\***: sum of squared residuals of the *i*th equation.
- **mse\***: estimated variance of the residuals (mean of squared errors) of the *i*th equation.
- **s2\***: estimated variance of the residuals ( $\hat{\sigma}^2$ ) of the *i*th equation.
- **rmse\***: estimated standard error of the residuals (square root of mse) of the *i*th equation.
- **s\***: estimated standard error of the residuals ( $\hat{\sigma}$ ) of the *i*th equation.
- **r2\***: R-squared (coefficient of determination).
- **adjr2\***: adjusted R-squared value.
- **maxiter**: maximum number of iterations.
- **tol**: tolerance level indicating when to stop the iteration.

## How to Cite

To cite the *sur* Zelig model:

Ferdinand Alimadhi, Ying Lu, and Elena Villalon. 2007. “sur: Seemingly Unrelated Regression,” in Kosuke Imai, Gary King, and Olivia Lau, “Zelig: Everyone’s Statistical Software,” <http://gking.harvard.edu/zelig>.

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Zelig: Everyone’s Statistical Software,” <http://GKing.harvard.edu/zelig>.

Kosuke Imai, Gary King, and Olivia Lau. 2008. “Toward A Common Framework for Statistical Analysis and Development,” *Journal of Computational and Graphical Statistics*, forthcoming, <http://gking.harvard.edu/files/abs/z-abs.shtml>.

## See also

The *sur* function is adapted from the `systemfit` library (Hamann and Henningsen 2005).

# Bibliography

Hamann, J. and Henningsen, A. (2005), *systemfit: Simultaneous Equation Systems in R Package*.