

## 0.1 chopit: Compound Hierarchical Ordered Probit for Survey Vignettes

The Compound Hierarchical Ordered Probit (CHOPIT) model corrects for “differential item functioning” or “interpersonal comparability” in ordinal survey responses. Given a self-assessment question (such as, “How healthy are you? Excellent, good, fair, or poor.”), different respondents may interpret the response categories in different ways, such that excellent health to one individual may be fair health to a hypochondriac. For each ordinal self-assessment to be corrected, the CHOPIT model requires one or more vignette question (such as a description of a hypothetical person’s health, followed by the same response categories as the self-assessment), and a set of associated explanatory variables for the respondent. The key assumption of the approach is that the thresholds (which determine how respondents translate their views into the response categories) have the same effect for different questions asked of the same respondent, but may differ across respondents; the model uses a parametric specification to predict the thresholds associated with an individual. The self-assessment and vignette questions may be taken from different surveys, so long as both surveys include the same explanatory variable questions to predict the thresholds. For ordinal data (without vignettes), see Section ??, Section ??, and Section ??.

### Syntax

```
> fml <- list(self = Y ~ X1 + X2,
              vign = cbind(Z1, Z2, Z3) ~ 1,
              tau = ~ X1 + X2)
> z.out <- zelig(fml, data = list(self = data1, vign = data2),
               model = "chopit")
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out, x1 = NULL)
```

### Inputs

In this hierarchical model, the `formula` and `data` inputs to `zelig()` are lists with the following structure:

- The `formula` is a list with three `formula` objects corresponding to:
  - `self`: Specifies the self-response question (Y) as a function of a set of explanatory variables.
  - `vign`: Specifies the vignette questions on the left-hand side as a matrix in the form `cbind(Z1, Z2, Z3)`.
  - `tau`: Specifies explanatory variables that constrain the cut points across both the vignette and self-response questions. These explanatory variables do not necessarily need to overlap with the set of explanatory variables specified in the `self`

formula, but must be observed in both the **vign** and **self** data frames, described below.

- The **data** argument is a list of two data frames with
  - **self**: A data frame containing the self-response question(s) specified in the **self** formula and associated explanatory variables listed in the **self** and **tau** formulas.
  - **vign**: A data frame containing the vignette questions specified in the **vign** formula and associated explanatory variables listed in the **tau** formula.

## Additional Inputs

In addition to the standard inputs, **zelig()** takes many additional options for compound hierarchical ordered probit regression, see **help(chopit)** and **?** for details.

## Examples

### 1. Basic Example

Setting up the formula as a list for the self-response, vignettes, and the cut points (drawn from both the self-response and vignette data sets).

```
> formula <- list(self = y ~ sex + age + educ + factor(country),  
+   vign = cbind(v1, v2, v3, v4, v5) ~ 1, tau = ~sex + age +  
+   educ + factor(country))
```

Attaching the sample data sets. The **free1** data correspond to the self-response data, and the **free2** data correspond to the vignette subset. Note that the variables specified in the **tau** formula must be in both data sets.

```
> data(free1, free2)  
  
> data <- list(self = free1, vign = free2)
```

Estimating parameter values for the CHOPIT regression:

```
> z.out <- zelig(formula, data = data, model = "chopit")
```

Setting values for the explanatory variables to their default values:

```
> x.out1 <- setx(z.out)
```

Simulating quantities of interest from the sampling distribution.

```
> s.out1 <- sim(z.out, x = x.out1)  
  
> summary(s.out1)
```

## 2. Simulating First Differences

Estimate the first difference in expected values between the average age (about 40 years old) and a 25 year old individual, with the other explanatory variables held at their default values:

```
> x.out2 <- setx(z.out, age = 25)

> s.out2 <- sim(z.out, x = x.out1, x1 = x.out2)

> summary(s.out2)
```

## 3. Conditional prediction

Conditional prediction generates expected values that are conditional on the observed self-response.

```
> x.out3 <- setx(z.out, cond = TRUE)
```

Since conditional prediction involves numeric integration, the procedure takes approximately one second per observation in `x.out3` on 64-bit R.

```
> s.out3 <- sim(z.out, x = x.out3)

> summary(s.out3)
```

## Model

This model has two sets of response variables, one for the self-assessment and one for the vignettes. Let  $Y_i$  be the observed ordinal self-assessment for respondents  $i = 1, \dots, n$ , and  $Z_{lj}$  be the ordinal vignette responses for individuals  $l = 1, \dots, L$  in the vignette subset for  $j = 1, \dots, J$  vignette questions, such that both  $\{Y_i, Z_{lj}\}$  take integer values  $k = 1, \dots, K$  corresponding to the same ordinal assessment response categories.

- The *stochastic components* are described by unobserved continuous variables,  $Y_i^*$  and  $Z_{lj}^*$ , which follows normal distributions with mean  $\mu_i$  and variance  $\sigma^2$  in the case of  $Y_i^*$ , and mean  $\theta_j$  and variance  $\sigma_j^2$  in the case of each  $Z_{lj}^*$ . Using the default identification mechanism, the variance  $\sigma^2$  for the self-assessment is fixed to 1. Thus,

$$\begin{aligned} Y_i^* &\sim N(\mu_i, 1) \\ Z_{lj}^* &\sim N(\theta_j, \sigma_j^2) \end{aligned}$$

such that each vignette response  $j$  has a scalar mean  $\theta_j$  and variance  $\sigma_j^2$  that does not vary over observations  $l$ . In cases where more than one self-response was administered

to the same subject, an additional random effect may be included in the distribution of the latent  $Y_i^*$  in the form

$$Y_i^* \sim N(\mu_i, 1 + \omega^2)$$

where the variance term is obtained via the proof described in Appendix A of ?.

The observation mechanisms that divide the continuous  $\{Y_i^*, Z_{lj}^*\}$  into the discrete  $\{Y_i, Z_{lj}\}$  are

$$\begin{aligned} Y_i &= k & \text{if } \tau_i^{k-1} \leq Y_i^* \leq \tau_i^k \text{ for } k = 1, \dots, K \\ Z_{lj} &= k & \text{if } \tau_l^{k-1} \leq Z_{lj}^* \leq \tau_l^k \text{ for } k = 1, \dots, K \end{aligned}$$

where the threshold parameters  $\tau$  vary over individuals  $\{i, l\}$ , but are subject to the following constraints within each individual:  $\tau^p < \tau^q$  for all  $p < q$  and  $\tau_0 = -\infty$  and  $\tau_K = \infty$ .

- There are three *systematic components* in the model.

- For the self-assessment component, let

$$\mu_i = x_i \beta$$

where  $x_i$  is the vector of  $q$  explanatory variables for observation  $i$ , and  $\beta$  is the associated vector of coefficients.

- In addition, the threshold parameters also vary over individuals in the self-assessment component as follows

$$\begin{aligned} \tau_i^1 &= v_i \gamma^1 \\ \tau_i^k &= \tau_i^{k-1} + \exp(v_i \gamma^k) \text{ for } k = 2, \dots, K \end{aligned}$$

where  $v_i$  is the vector of  $p$  explanatory variables for observation  $i$ , and  $\gamma^k$  for  $k = 1, \dots, K$  are the vectors of coefficients associated with each categorical response. Thus, the threshold parameters vary over individuals since  $v_i$  vary, and over response categories since the  $\gamma^k$  vary over the threshold parameters.

- Similarly, the threshold parameters vary over individuals in the vignette component as follows

$$\begin{aligned} \tau_l^1 &= v_l \gamma^1 \\ \tau_l^k &= \tau_l^{k-1} + \exp(v_l \gamma^k) \text{ for } k = 2, \dots, K \end{aligned}$$

where  $v_l$  is a vector of  $p$  explanatory variables for observation  $l$  in the vignette subset, and  $\gamma^k$  are restricted to be the same  $\gamma^k$  used to parameterize the threshold parameters for the self-assessment component.

As ? note, the interpersonal comparability of responses (or response consistency) is achieved by constraining  $\gamma^k$  to be the same in both the self-assessment and vignette components of the model. Note that the variables included in  $v_i$  and  $v_l$  are the same, but the observed values of those variables differ across the vignette and self-response samples.

## Quantities of Interest

- The expected value (`qi$ev`) for the CHOPIT model is the expected value of the posterior density for the systematic component  $\mu_i$ ,

$$EV = E(\mu_i | x_i) = x_i\beta$$

given draws of  $\beta$  from its sampling distribution.

- The first difference is the difference in the expected value of the posterior density for the systematic component  $\mu_i$  given  $x_1$  and  $x_0$ :

$$FD = E(\mu_i | x_1) - E(\mu_i | x_0).$$

- In conditional prediction models, the conditional expected values (`qi$cev`) are the expected value of the distribution of  $\mu_i$  conditional on the observed self-assessment response  $Y_i$ , where

$$P(\mu_i | \tau_i, \beta, x_i, Y_i) = \prod_{k=1}^K [\Phi(\tau_i^k - \mu_i) - \Phi(\tau_i^{k-1} - \mu_i)] \times N(x_i\beta, x_i\widehat{V}(\widehat{\beta})x_i' + \widehat{\omega}^2)$$

given the simulations of the threshold parameters calculated above, draws of  $\beta$  from its sampling distribution, and the estimated variance-covariance matrix for  $\widehat{\beta}$ .

## Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run `z.out <- zelig(..., model = "chopit")`, then you may examine the available information in `z.out` by using `names(z.out)`, see the estimated parameters by using `z.out$par`, and a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output object `z.out`, you may extract:
  - `par`: the maximum likelihood parameter estimates for  $\widehat{\gamma}^k$  for  $k = 1, \dots, K$  response categories,  $\log(\widehat{\omega})$  (if estimated),  $\log(\widehat{\sigma})$  (if estimated),  $\log(\widehat{\sigma}_j)$  for  $j = 1, \dots, J$  vignette questions,  $\widehat{\theta}_j$ , and  $\widehat{\beta}$ .
  - `chopit.hessian`: the estimated Hessian matrix, with rows and columns corresponding to the elements in `par`.

- `value`: the value of the log-likelihood at its maximum
  - `counts`: the number of function and gradient calls to reach the maximum.
  - `formula`: the formula for `self`, `vign`, and `tau` selected by the user.
  - `call`: the call to `zelig()`.
  - `...`: additional outputs described in `help(chopit)`.
- Typing `summary(z.out)` provides a useful summary of the output from `zelig()`, but no items can be extracted.
  - From the `sim()` output object `s.out`, you may extract quantities of interest arranged as matrices indexed by simulation  $\times$  `x`-observation (for more than one `x`-observation). Available quantities are:
    - `qi$ev`: the simulated expected values for the specified values of `x`.
    - `qi$fd`: the simulated first difference in the expected values for the values specified in `x` and `x1`.
    - `qi$cev`: the simulated conditional expected value given `x`.

## How to Cite

To cite the *chopit* Zelig model:

Kosuke Imai, Gary King, and Olivia Lau. 2007. "chopit: Compound Hierarchical Ordinal Probit Regression for Survey Vignettes" in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software," <http://gking.harvard.edu/zelig>

To cite Zelig as a whole, please reference these two sources:

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Imai, Kosuke, Gary King, and Olivia Lau. (2008). "Toward A Common Framework for Statistical Analysis and Development." *Journal of Computational and Graphical Statistics*, Vol. 17, No. 4 (December), pp. 892-913.

## See also

The CHOPIT model is part of the `anchors` package by Jonathan Wand, Gary King, and Olivia Lau (?). Advanced users may wish to refer to `help(chopit)`, as well as ? and ?.